GROUPED OBJECTS AS A CONCRETE BASIS FOR THE NUMBER IDEA

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The purpose of this article is to give the results of an experimental investigation of number perception in children and adults, and to discuss the bearing of the results of the investigation upon the early development of the number idea in the child.[1] At the outset of the experiment the question to which an answer was sought was the following: What is the difference between the number of objects which may be grasped together in a single act of the attention by an adult and by children of various ages? The question may be illustrated from the field of vision. If there are a number of similar objects in a small group one finds it possible to distribute his attention at a given moment over a certain number of these objects, but if the number is increased he soon finds a limit beyond which it is impossible to grasp all the objects simultaneously. That is, if he attempts to include more in his attention he will find that the collection begins to break up into two or more groups each of which is small enough to be recognized. He then finds an oscillation of attention between groups. The number of objects which can be held simultaneously in a single act of attention by an adult is fairly uniform for different people and averages about five or six. The number of objects which can be grasped in this way is said to determine the scope of attention.

The question with which the experiment was begun, was then: How far do children differ from adults in this elementary capacity called the scope of attention? The assumption which underlay this investigation was that the answer to the question would have some bearing upon the relation between the number perception of

[1] For a detailed account of the experiment and its result see Frank N. Freeman, "Aufmerksamkeitsumfang und Zahlauffassung bei Kindern und Erwachsenen," in *Arbeiten aus dem Institut für Psychologie und experimentelle Pädagogik*, Band I; Alfred Hahn, Leipzig.

children and that of adults. The perception of a series of objects does not necessarily involve number perception, it is true. That is, a person may be able to compare two groups of objects and say that one group contains more objects than the other without definitely counting the number of objects in either group. Again we may test whether or not a person has correctly grasped a group of objects by requiring him to draw them. This might merely be an evidence that he has retained a visual image of the objects and of their arrangement. Although the ability to grasp a series of objects in the attention does not necessarily involve a developed number perception, it does undoubtedly underlie number consciousness. From any point of view, therefore, a study of the scope of attention will throw light on number ideas.

In order to determine the scope of attention in its simplest form it is necessary to present objects which do not have any definite form of grouping. The apprehension of a number of objects simultaneously under these circumstances requires that each object be regarded as a separate unit. If in even this first case one can arrange the objects in a group in his mind he can grasp very many more than the five or six which mark the limit of the scope of attention. In order to measure the scope of attention under relatively very simple conditions without grouping so far as the external objects were concerned, the objects were arranged in a horizontal line with an equal distance between them as in Series I, Fig. 1. Later, in order to test more complex perception, which has a more direct relation to the number idea, the objects were arranged in various kinds of groups. The chief forms of grouping which were used are shown in Fig. 1. The results of tests with these groups showed that greater difference in general exists between the children and adults in the perception of the grouped objects than in the perception of the ungrouped objects.

The method of presenting the objects was to expose upon a large screen and for very brief intervals of time the desired number of circles of light upon a dark background. The room was kept in dim light and the spots of light upon the screen were enough brighter than the background to be clearly seen. The projection apparatus was on the opposite side of the screen from the child and

was, therefore, out of sight. It consisted in a stereopticon lantern in which were inserted cards with holes of various numbers and arrangements punched in them. The exposure was made by allowing a pendulum upon which was fixed a large screen having in it an opening of variable size to swing through the beam of light. By adjusting the size of this opening the length of time of the expo-

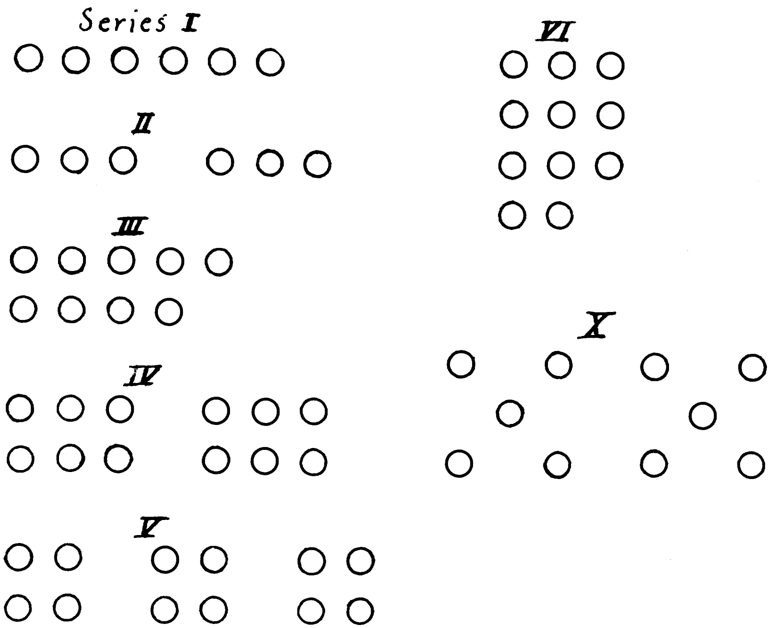

FIG. 1.—Illustrations of forms of grouping used in the experiment

sure could be regulated. The results of the experiment may be summarized as follows.

The persons who served as subjects of the experiment included both children and adults. The ages of the children ranged from six to fourteen years, but the children below the age of eight gave such incomplete and varying results that no positive conclusions could be drawn regarding their perception of number. This was probably due to their inability to grasp the stimulus in the brief time during which it was exposed. The differences which were found between the eight- to ten-year-old and the twelve- to fourteen-

year-old children can, however, be extended, without serious danger of error, to the younger children.

The first fact which was brought out was that less difference exists in the scope of attention between adults and children than might be expected. If we place the scope of attention of the adults and older children at about five, that of the younger children would be not less than four. That is, up to and including four objects, the younger children could judge the number as correctly as adults. Later results make it clear that there are differences of a marked character between adults and children; this difference in the number of objects which can be grasped simultaneously may account in some measure for the difference in the capacity of children and adults to deal with number, but it can account for only a small part of the difference between the mature and immature individual. Though there is some difference in this elementary capacity yet the greater part of the difference must be ascribed to the more complex process of organizing the impressions which are received and not to the simple process of grasping the impressions themselves. This was brought out clearly by the fact that as the numbers which were shown increased beyond the limits of the scope of attention, the correctness of judgment of the children fell off very much more rapidly than was the case with adults. This introduces us to the second main result of the experiment.

Though the difference in the scope of attention between adults and children was not great, there was a marked difference in the ability to perceive and correctly judge grouped objects which extended considerably beyond the scope of attention. A marked tendency on the part of adults to organize their perceptions into regular groupings was manifested even in the case of the objects which were in reality ungrouped. That is, an adult was very apt to divide a series of objects equidistant from one another and arranged in a horizontal line into several subgroups of from two to four each. By this process the whole number could be very much more easily managed. This, then, accounts for the fact above mentioned that adults could correctly judge numbers which extended beyond the number which they could grasp directly, even if the number extended much beyond the scope of attention.

When it came to the judgment of objects which were objectively grouped, the same result appeared. Adults and older children nearly always noticed the way in which the objects were arranged. A younger child, on the other hand, was very much slower in seeing that the objects were arranged in the form of a square or in groups of three, five, or six as the case might be, and in many cases the younger children failed entirely to grasp the arrangement. Furthermore, when they did arrange the objects into groups they often put them in a far-fetched and unnatural arrangement. For example, it occurred several times that a child would see two groups of five as in Series X, in the following way: One horizontal row of four, one horizontal row of two, and a second horizontal row of four. Or the groups of four objects, Series V, might be seen as groups of two in horizontal rows. Obviously, then, the younger children were very deficient in the way in which they organized the impressions into regular forms of grouping.

This organization of objects into regular forms is very closely related to number processes, and, as will be shown later, training in this form of perception is one of the best means of giving the child a grasp of the simpler number operations. The child's ordinary experience gives some basis for education in this organization of his impressions into regularly arranged groups. Most of the things with which he has to deal are so arranged. The plates on a table, for example, are symmetrically arranged. The panes in a window have a regular arrangement. The desks in school are arranged in rows with so many in each row. The trees on a street are planted in regular order. Thus his everyday experiences lead the child to the notion of grouping objects and of dealing with them as groups made up of certain numbers of individuals instead of leaving him with the tendency to look at individuals separately. This informal education, however, might very well be supplemented more extensively by formal training in the recognition of grouped objects.

Some of the applications of these general principles to the training of the child in number perception and in the use of number may be made.

The method of using groups of individual objects to illustrate

various numbers and their combinations was formerly employed by means of the old-fashioned abacus. This has largely gone out of use in the United States, but its place has been taken in Germany by somewhat more elaborate forms of apparatus. Introductory number work which consists in counting groups of objects has been largely replaced in the United States by the use of measurement as the first stage of arithmetic. This is perhaps due in large measure to the idea that only those units are suitable for illustrating number which are absolutely identical. It is obvious that units of measurement are more nearly identical than separate individual objects. The belief, however, that it is necessary to use identical units is entirely without foundation, and overlooks the fact that number is abstract and is not dependent upon the other characteristics of objects. We may choose to regard any object whatever as a unit and may count it with other units of different concrete nature. In fact it is advantageous at times to use different kinds of objects in order to bring out the fact that the number relations of objects are different from their other relationships. For example, it is perfectly possible to count together twelve chairs and six tables and say we have eighteen pieces of furniture.

The abacus, however, was not especially well suited to this purpose because of the fact that it did not permit of arranging the objects in various forms of grouping. The "reckoning machines" which are used in Germany are so constructed that the objects may be arranged in groups of two, three, four, five, etc. In addition they are so constructed that a group may be made up of two subdivided groups. The subdivision may be made by a line between two subgroups or by having a part of the large group in one color and part in another. Whatever the device used, the aim is conveniently to arrange objects in different groups and subgroups. It is not necessary, however, to have an apparatus for this purpose. The same aim can be attained by the use of the blackboard. The main point to be established is the value of the method.

Other forms of concrete experience are largely used in developing the number idea in the child. Each of these has certain characteristic advantages and disadvantages. The first method

by which the child is introduced to the idea of number is ordinarily that of counting. The child learns to count before he comes to school. Some of the forms of counting, however, obviously do not involve any idea of number. The child may say over the number names without applying them to any objects in particular, just as he would say over the A, B, C's. He may even say over the names of the numbers and at the same time point to objects, and still not have a clear idea of the fact that the final number which he reaches refers to the whole group of objects rather than to the last one in the series. That is, his idea may be of the ordinal type, first, second, third, etc., in which the number refers to a position in a series rather than to a whole group. Counting, therefore, serves as an introduction to number, but is not itself self-sufficient for the complete development of the idea. Some form of grouping must be added.

The process of measurement which has been extensively adopted instead of counting differs from counting or the perception of grouped objects in that the unit is not a concrete object but is a fraction of a whole quantity. For example, when we say that a pole is five feet long, or that a pail contains five quarts of water, the individual foot, or the individual quart, is not sharply set off from the other units of the whole quantity. The unit is itself represented, it is true, by a measure which in the illustrations would be a foot rule or a quart measure, but in the recognition of the concrete quantity which represents a multiple of the unit, the unit itself does not appear. Measurement, then, has its particular advantage and its particular disadvantage. The advantage is that the attention is naturally directed to the whole quantity which the number represents, as contrasted, for example, with the tendency in counting to see each element of the group separately. The disadvantage, however, is that the attention is not naturally directed to the units of which the quantity is composed. The illustration may be further developed in connection with the concrete representation of a simple number operation. If we divide an area or a quantity of any sort into two fractional parts, we may easily perceive that the whole quantity is equal to the sum of the two fractional quantities. Our attention is not, however, directed

to the actual number relations between the two fractional quantities and the whole, as it is when a group of objects is divided into two subgroups.

It is precisely at this point that the use of objects in various forms of grouping is suited to represent in a concrete way the number operations. If we have a group of five objects we may fix our attention upon this group as a whole, or we may divide the group into two parts and fix our attention upon a group of three objects by itself and upon a group of two objects by itself. This very clearly brings out the numerical fact that a group of five objects may be composed of a smaller group of three and a group of two. We here have combined the view of the whole quantity as represented by the total number and at the same time the thought of the individual units of which the whole group is composed.

In conclusion, we may further illustrate the way in which grouped objects may be used to form the concrete basis for the idea of the fundamental number operations. Suppose that we assume that the child is able to grasp at once four objects. He may then comprehend that a group of four objects may be made up of two groups of two objects each, and when he has grasped this fact he has comprehended the real nature of division. He may not even know the names of the numbers. Of course in order to use this idea in a practical way it is necessary that he have a command of the number names and of the figures which represent them. But the mere manipulation of the figures is not evidence that he grasps the idea, and if he has not grasped the idea the manipulation of the figures is meaningless to him. In the same way subtraction may be grasped by abstracting one object from a group of four and perceiving that the group of three and the group of one are together equivalent to the group of four. The correlative ideas of multiplication and addition are illustrated by reversing the process.

This process of dealing with groups may be extended by using multiples of the groups which are within the child's grasp. That is, if he grasps three separate objects as belonging together he may also grasp together three groups of three objects each. He thus

has at his command a much larger number of objects and he can learn to comprehend them in precisely the same manner that adults comprehend numbers which are beyond their grasp. The decimal system is really a means of extending our notion beyond the numbers which we can hold in attention. If the number name *one hundred* means anything to us beyond a name, it means an extension in a certain orderly way of numbers which are beyond our grasp, on the analogy of the extension which may be made within our concrete apprehension. The child may thus be introduced, not merely to the fundamental forms of number operation, but also to the ideas of the larger numbers.